quick study

Swimming in the desert

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Physicists and colleagues from various disciplines are just beginning to study how organisms and robots move within and on granular media. An old theory from fluid mechanics can help guide them.

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Locomotion, the movement of a body from place to place via self-deformation and interaction with an environment, is so ubiquitous in the natural world that many people don't give it a second thought. But locomotion challenges scientists to understand the dynamics of biological and physical systems. Furthermore, the substrates that animals move within—examples include air, water, sand, and mud—can be deformable; for the cases of sand and mud, scientists still don't have a fundamental description of substrate rheology. Organisms have an enormous number of degrees of freedom, and their numerous muscles, tendons, and skeletal elements are hierarchically organized and controlled by complex nervous systems. Despite those complexities, biologists, mathematicians, engineers, and physicists have made great strides in understanding movement.

Traditionally, movement on rigid ground or in flowing fluids like air and water has been scientists' focus. But many terrestrial animals move on the surface of and even within substrates that are neither ideal solids nor ideal fluids. One such substrate is sand. Like other granular media, sand is a collection of solid particles that interact through dissipative, repulsive contact forces. Granular media can flow as fluids do, but they can also hold their shape like solids. Sand, in particular, is home to many arachnids, reptiles, and mammals that run rapidly across its surface to hunt for prey. Some, such as the sandfish lizard shown in figure 1, even dive into sand and move within it to escape heat and predators.

A look below the surface

Despite hundreds of years of study, scientists do not yet have a comprehensive understanding of the flow and effective solidification of granular media subject to intrusion by a body or leg. One reason is the complexity of grain response to localized forces. For example, granular materials can support loads through the formation of complex force-chain networks, exhibit compressible flow with heterogeneous regions of density and pressure, or even act like gases described by the kinetic theory. (For more, see the article by Anita Mehta, Gary Barker, and Jean-Marc Luck, Physics Today, May 2009, page 40, and the Quick Study by Jackie Krim and Bob Behringer, Physics Today, September 2009, page 66.)

Figure 1. The sandfish is native to north Africa and the Middle East. (Courtesy of John Toon.)

In 2009 Ryan Maladen and the three of us performed the first detailed explorations of locomotion within granular media; as shown in figure 2a, we used high-speed x-ray imaging to record the movement in sand of a 10-cm-long lizard, the sandfish *Scincus scincus*. In those experiments we discovered that when it is below the surface, the lizard tucks its legs to its sides and swims forward by means of a sinusoidal body undulation traveling from head to tail. We were surprised to discover that the wave shape resembled that of the smaller, 1-mm-long nematode worm *Caenorhabditis elegans* swimming in a viscous fluid. However, the forward speed of the sandfish in body lengths per second was greater than that of *C. elegans*. Furthermore, the sandfish moved nearly 0.4 body lengths per undulation, double the rate achieved by *C. elegans.*

In trying to explain those observations, we faced a problem: We had no means to calculate the interaction of the sandfish with the granular medium. Further, we didn't know if the animal was swimming in a solid, a fluid, or a complex combination of both. In part because the sandfish looked so much like *C. elegans*, we posited that the granular medium surrounding a continuously moving locomotor would behave like a fluid. With nothing else to try, we appealed to an old theory from fluid mechanics.

A sandfish superposition principle

The partial differential equations describing fluid motion—the Navier–Stokes equations—are complex and nonlinear. So for more than 60 years, researchers have used a simplified theoretical approach called resistive force theory (RFT) to model undulatory and flagellar propulsion in fluids at low Reynolds number *Re*—that is, for motion in which inertial forces are small relative to viscous forces. The theory, introduced by G. I. Taylor and by James Gray and G. J. Hancock, is based on the assumption that the forces $F_\perp$ and $F_\parallel$ acting perpendicular and parallel to the axis of an infinitesimal body element are independent of the position and movement of other body elements. Note that the ratio of $F_\perp$ to $F_\parallel$, the so-called drag anisotropy, must be greater than unity for forward propulsion. At low *Re*, the inertial terms in the Navier–Stokes equations can be neglected, and thus, at constant swimming speed, the sum of all the independent, superposed forces on the organism vanishes. Given that simplification, one can calculate swimming speeds in terms of, for example, the speed at which the undulation travels down the organism.

To do the calculation, one needs to know how $F_\perp$ and $F_\parallel$ depend on the orientation, relative to the movement direction, of a submerged body such as a cylinder (see figure 2b). For low *Re* fluids, those forces can be calculated theo-

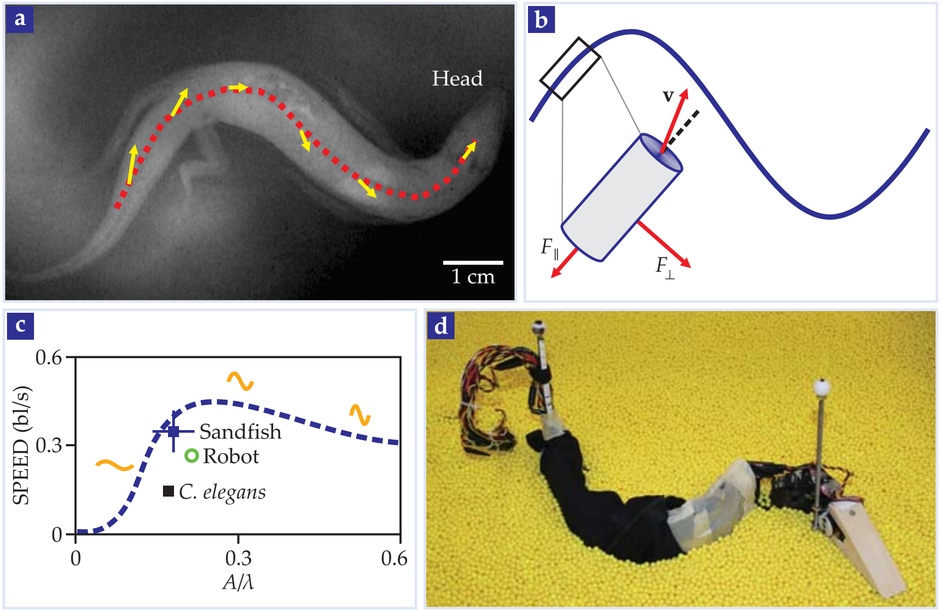


Figure 2. Locomotion within dry granular media. **(a)** This top-view x-ray snapshot shows a sandfish lizard swimming in a medium of small glass particles. Yellow arrows are estimates of the velocity of small elements of the sandfish based on resistive force theory. The overall motion is to the right. (Courtesy of Sarah Sharpe.) **(b)** In RFT, the forces on organism elements are assumed to be independent; here an element is represented as a cylinder. One needs to know how the forces parallel ($F_\parallel$) and perpendicular ($F_\perp$) to the cylinder axis depend on the orientation of the cylinder relative to its velocity **v**. **(c)** Swimming speeds in body lengths per second are shown for the sandfish, the 55-cm-long robot seen in **(d)**, and the worm *Caenorhabditis elegans*, as a function of the amplitude (A) and wavelength (λ) indicated by the yellow curves. The sandfish achieves almost the optimal motion predicted by RFT (dashed curve). (Panel c adapted from R. D. Maladen et al., *J. R. Soc. Interface* **8**, 1332, 2011.)

retically. But for our study of motion in a granular medium, we introduced a cylinder to represent a small element of the sandfish's body and determined the forces experimentally.

We discovered some interesting ways in which horizontal drag forces in granular media differ from those for low *Re* motion in fluids. For example, at low *Re*, fluid forces are proportional to speed, but in granular media, forces are insensitive to speed—at least at the speeds of 50 cm/s or less that are characteristic of sandfish locomotion. The reason for the constancy is that for sandfish-speed motion in granular media, particle inertia is negligible; forces result mainly from friction between body elements and the particles around them and from particle–particle friction. Here's another interesting difference: In fluids, horizontal drag forces on a small body are independent of the body's depth beneath the surface, but in granular media, such forces are proportional to depth because of a hydrostatic-like pressure.

We found similar behavior for $F_\parallel$ as a function of the orientation of the cylinder axis relative to the velocity when we looked at motion in granular media and at low *Re* motion in fluids. But the behavior of $F_\perp$ was different; in granular media, the force rose more rapidly at shallow angles of attack (small angles separating cylinder axis and velocity). The mechanism underlying that different behavior is unclear, but we hypothesize that it is related to a solid-like pileup of grains near the surface.

An efficient swimmer

When we summed all the forces acting on the sandfish and set that sum to zero, we were surprised to find that RFT predicted the speed of the animal to within 20%. The agreement indicates that our picture of a sandfish swimming in a granular fluid is reasonable; later simulations revealed that the fluid is localized to a small region around the body. Moreover, RFT showed that the larger $F_\perp/F_\parallel$ in granular media accounted for the fact that the sandfish advances a greater number of body lengths per undulation than does *C. elegans*.

The theory also predicted a tradeoff between the increasing thrust generated when the undulation amplitude increases and the reduction of speed that ensues because the "wavelength" of the animal is thereby reduced. That tradeoff results in an optimal undulation for the most rapid swimming and the least energy consumed per unit distance moved. As shown in figure 2c, the sandfish swims almost optimally. We also tested the utility of RFT in granular media with the robot sandfish shown in figure 2d. Because of the robot's discrete construction, it can't swim as efficiently as the sandfish. But once that structure is taken into account, RFT does a good job of predicting optimal robot locomotion.

Future experimental studies, in combination with granular-media simulations, may shed more light on why RFT works well in granular media and where it fails. Potentially, such studies will also give insights about the universality of force relationships and their mechanistic basis. We have, for example, recently discovered that RFT predicts motion on the surface of granular media; that work, which involved robotic models, is briefly described in the online version of this Quick Study.

The information that scientists learn about natural locomotion should also be applicable to human-made devices operating in sandy terrain—for example, extraterrestrial rovers or robots that help at disaster sites. Those devices can certainly benefit from improved maneuverability. The Mars rovers, for example, have a top speed of 5 cm/s and occasionally suffer debilitating loss of traction.

Additional resources

▶ S. Vogel, *Life in Moving Fluids: The Physical Biology of Flow*, 2nd ed., Princeton U. Press, Princeton, NJ (1994).

▶ J. Gray, G. J. Hancock, "The propulsion of sea-urchin spermatozoa," *J. Exp. Biol.* **32**, 802 (1955).

▶ R. D. Maladen et al., "Undulatory swimming in sand: Subsurface locomotion of the sandfish lizard," *Science* **325**, 314 (2009).

▶ R. L. Hatton et al., "Geometric visualization of self-propulsion in a complex medium," *Phys. Rev. Lett.* **110**, 078101 (2013).

▶ B. Rodenborn et al., "Propulsion of microorganisms by a helical flagellum," *Proc. Natl. Acad. Sci. USA* **110**, E338 (2013).

▶ C. Li, T. Zhang, D. I. Goldman, "A terradynamics of legged locomotion on granular media," *Science* **339**, 1408 (2013). ■

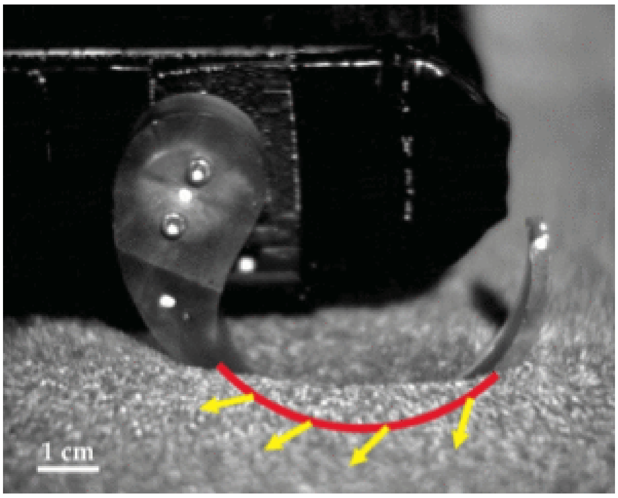
Supplemental material

Movement on granular media

The above Quick Study illustrates how a simple model of fluid mechanics, resistive force theory (RFT), could be applied to the locomotion of the sandfish lizard as it swims in sand. The key idea behind RFT is that the force acting on one small element of a sandfish is independent of all the other forces acting on the animal.

With confidence in that independent superposition principle, Tingnan Zhang and two of us (Li and Goldman) decided to test RFT in another situation: movement on the surface of granular media. That kind of locomotion is prevalent in the natural world; many animals walk and run on desert and beach sand. It is also of great engineering significance. The discipline of terramechanics, developed during the past 60 years, has helped engineers improve the mobility of both large-wheeled and tracked vehicles on deformable surfaces like soil and sand. Still, there is much room for further improvements. Terramechanics applies to wheels and tracks whose contacts with the terrain are comparable to those of large plates that indent the surface, and it uses many phenomenological fitting parameters specific to wheel and track geometries. It is thus not clear how to apply the lessons of terramechanics to legged locomotors.

To test whether RFT works for locomotion on the surface of a granular medium, we performed experiments on rigid C-shaped legs such as the one shown below (yellow arrows denote movement directions of leg elements). We had previously found such legs to be useful in robots walking on a variety of granular terrains.



As the robot walks, its legs rotate. In one experiment, we measured the net vertical and horizontal forces on a leg rotating about a fixed axis through a granular medium and determined how those forces depend on the rotation angle. We also obtained forces on infinitesimal leg elements. The force relations we found were more complicated than those for the sandfish lizard, which swims essentially horizontally within a granular medium. For the robot legs, which execute significant vertical motion, leg-element forces depended on both leg-element orientation and movement direction, not just on the angle between the two. The source of the additional complexity is gravity, which breaks vertical translation invariance. Also, because of the hydrostatic-like pressure in granular media, the forces are proportional to depth, as they are for the swimming sandfish. Remarkably, however, we discovered that the force relations were basically the same—to within a scaling factor that depended on the substrate strength—across a variety of granular media of different particle properties, including size, density, shape, friction, uniformity, and compactness. Integrating the leg-element forces over the entire leg, we found that RFT did a good job of predicting the net force on the robot legs, typically coming to within 5% of our measurements.

Finally, for the toughest test, we used RFT to try to optimize leg shape on a running robot. Below you can see a robot running on dirt; the robot we used for our optimization studies was similar in shape but quite a bit smaller.

The answer to the optimization question has potential engineering applications, and also biological relevance.

Solving the dynamics of on-ground locomotion is more involved than for sandfish swimming. For the sandfish, the balance of thrust and drag determines the animal's speed. Surface locomotion is more complicated in part because the legs do not constantly maintain ground contact; as a result, ground reaction force and body weight are not always instantaneously balanced. In addition, we could not assume zero body inertia as we did for the sandfish, which swims fully immersed in a dissipative medium. To address those complications, we combined RFT with a multibody dynamic simulation. To experimentally vary leg shape, we used a 3D printer to fabricate arc-like legs of different curvatures.

Our model and simulation predicted the speed of the robot with an accuracy of 20% or better for a wide range of leg curvatures and stride frequencies. Moreover, as the following graph shows, they also predicted the optimal leg curvature for the robot to move fastest at given stride frequencies on the granular medium.